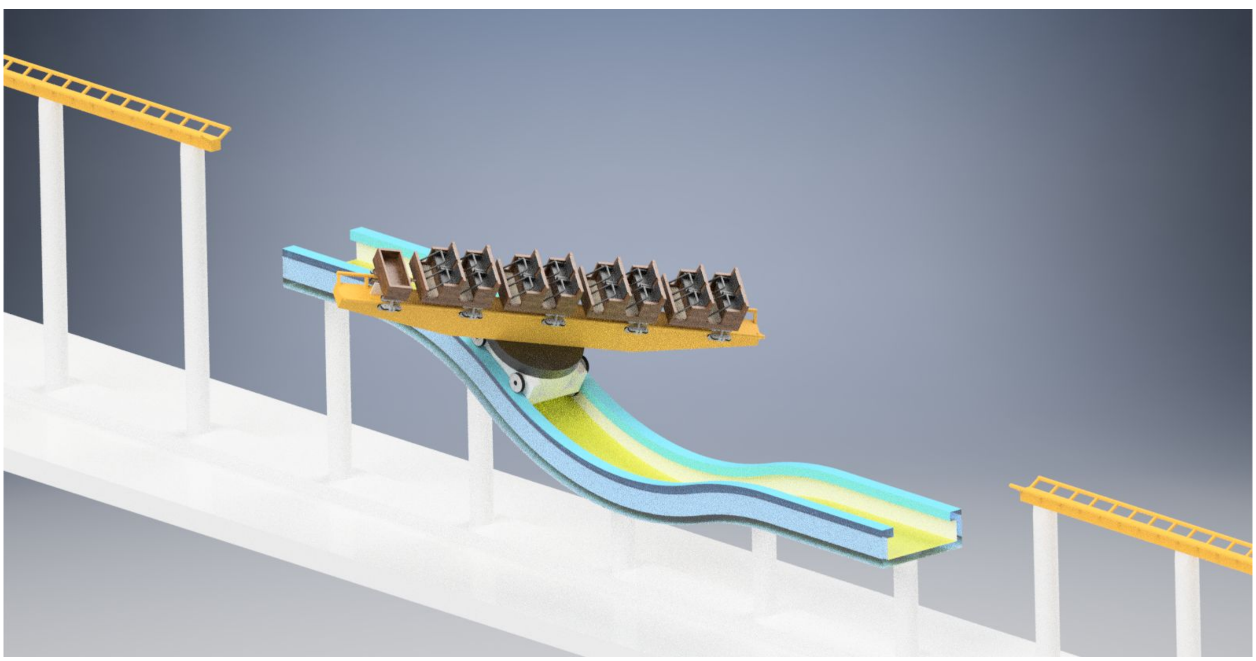
Rotary Table Motion Track
Technical Analysis

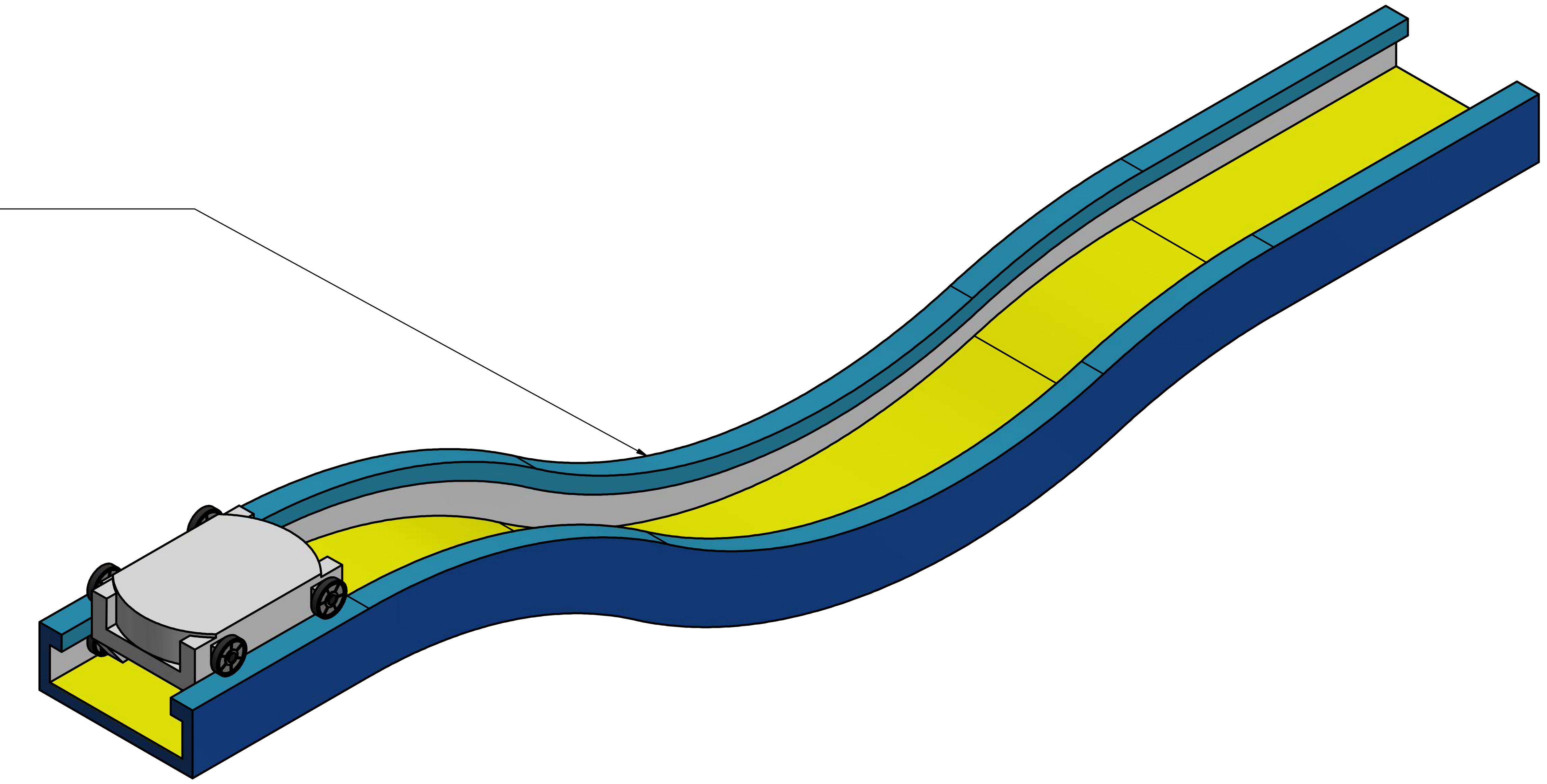
Team 10

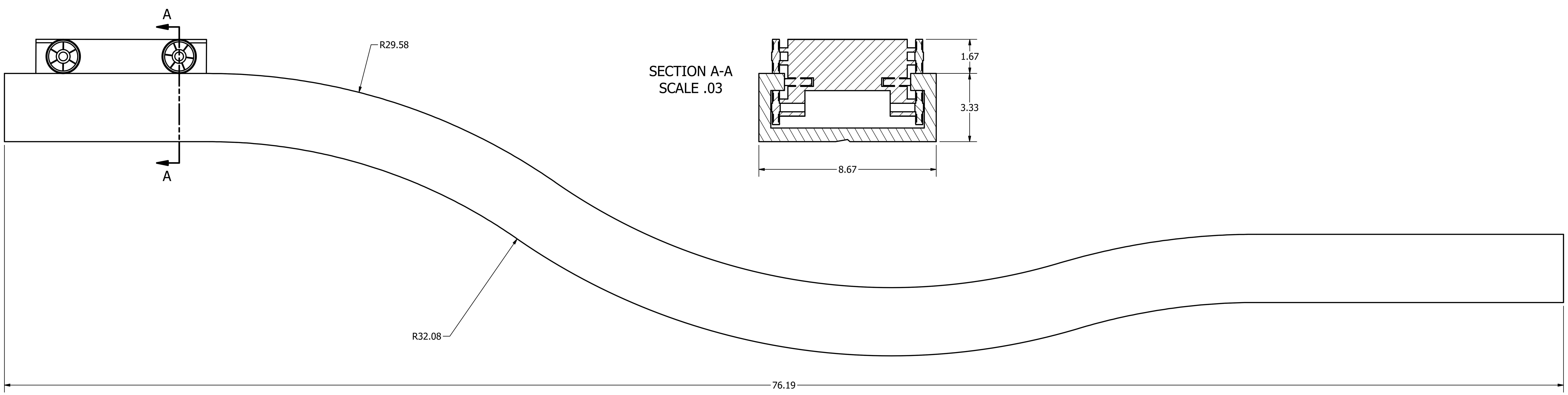


Based on the current size of the mechanism chassis, the minumum radius of track in the z-direction (changing the pitch of the train) is 43'-0". If the mechanism's chassis is shortened or its contact point to its respective track is raised while other variables remain the same, this minimum radius will get larger.

It should be noted that this is only true for when the mechanism's track is concave (turning upward). When it is convex (turning downward), the only limitation is the height of the roller coaster's platform to the wheels. However, this distance is quite large and it is unrealistic that the mechanism will ever pitch so steeply.

This drawing serves as a realistic depiction of where and how the general body and locations of parts of the mechanism are located. The size and scale of parts are subject to change based on more accurate calculations of forces and necessary resistances.

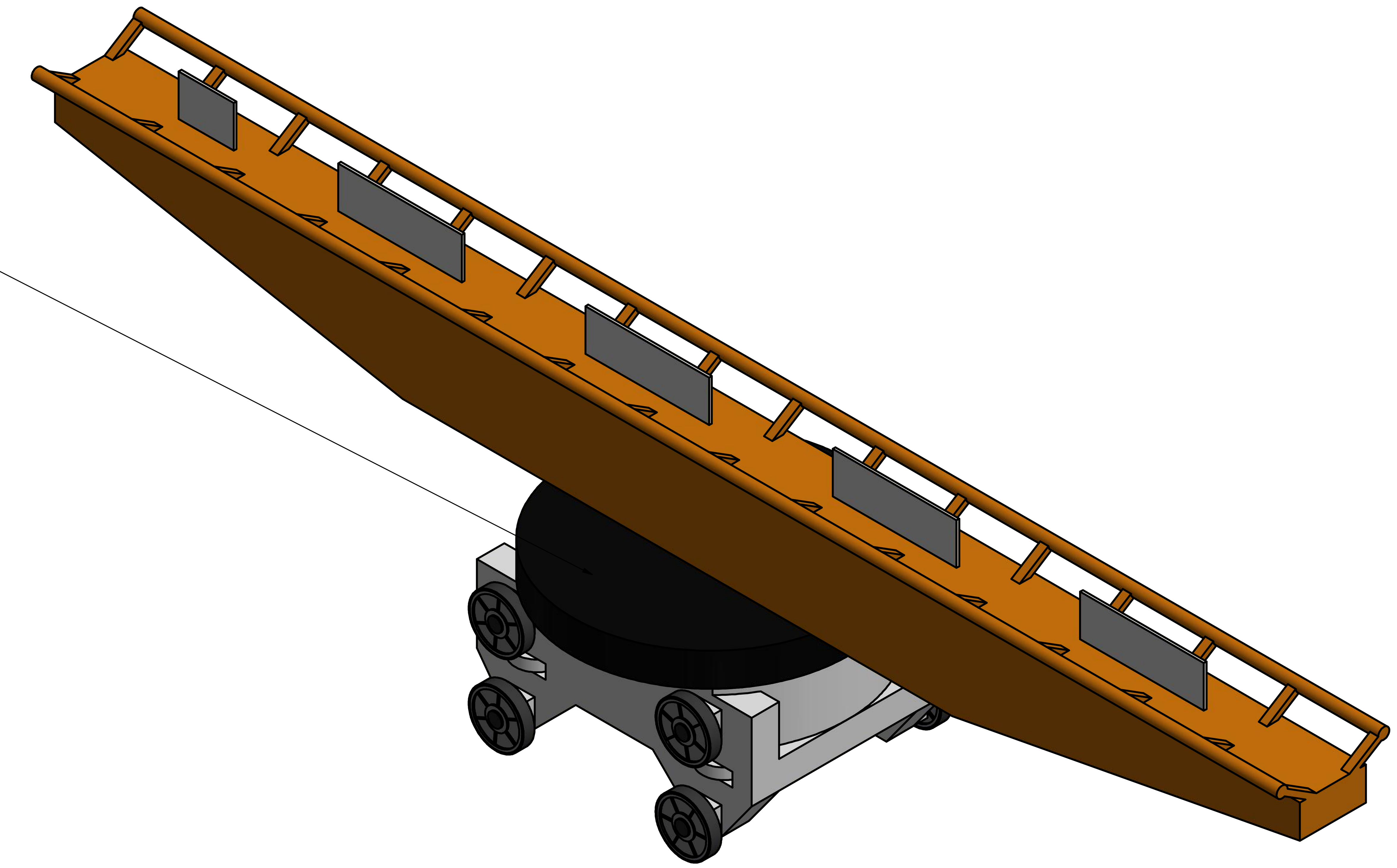


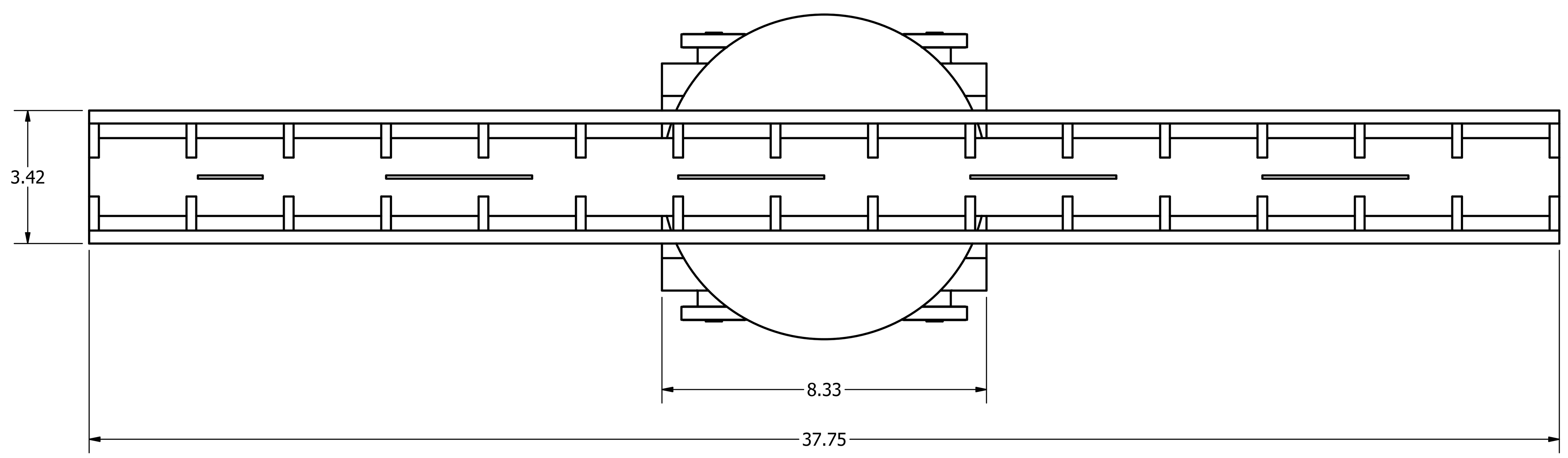


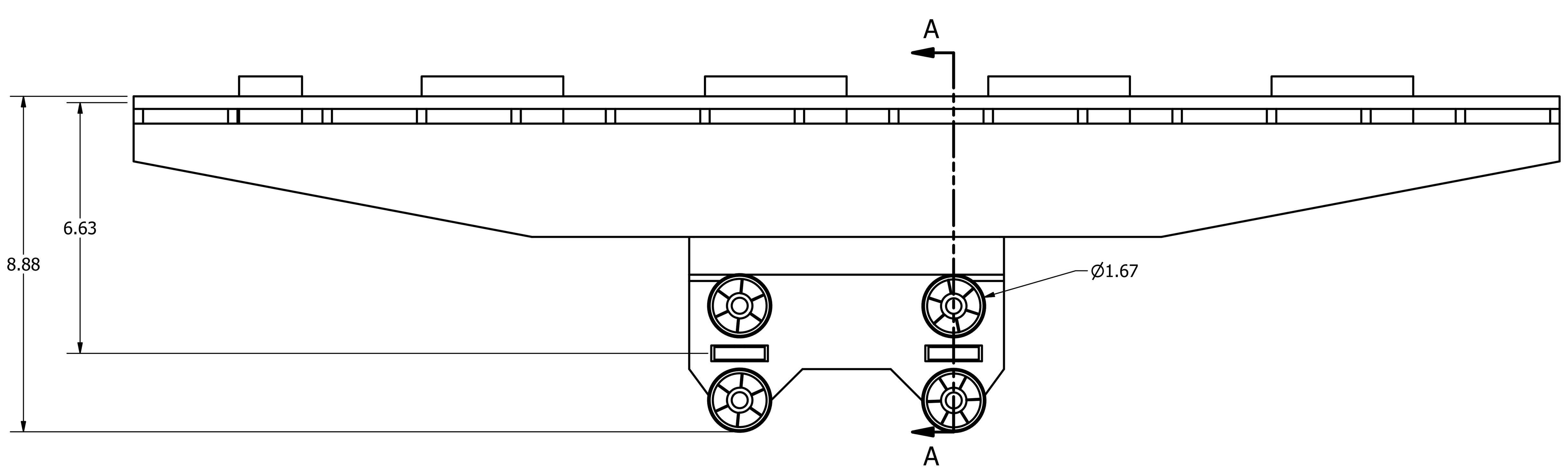
SECTION A-A
SCALE .03

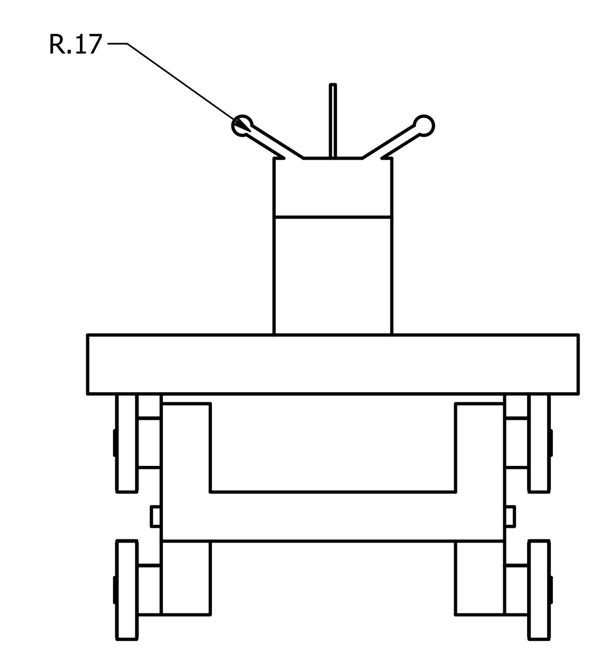
DRAWN TEAM 10	5/1/2019	
CHECKED		TITLE
QA		Glacier Gorge Novel Technology
MFG		
APPROVED		SIZE D / DWG NO Mechanism Track and Chassis / REV
		SCALE .02 / SHEET 1 OF 1

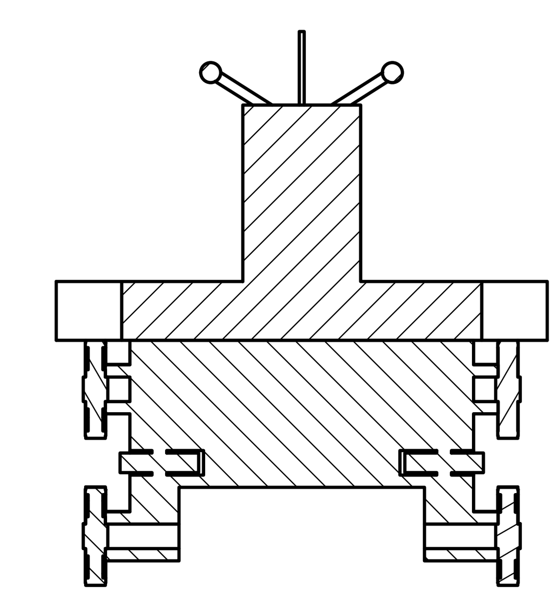
This is a rotary table, powered through a separate motor. It is mounted to both the track and the chassis on both of its individual plates, and connected through a pivot point in the middle. There are multiple smaller wheels located all around the body for added stability. A demo of this is exemplified in the attached videos.









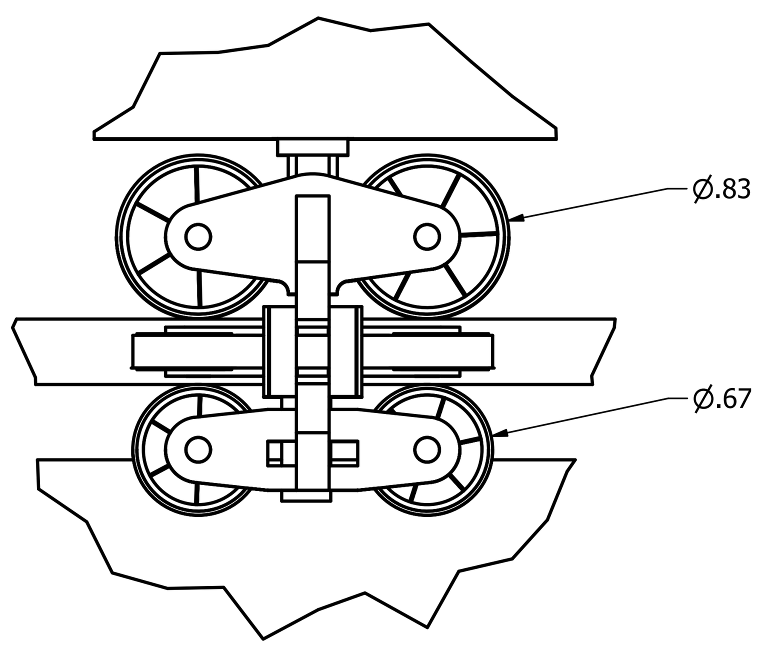


SECTION A-A
SCALE .03

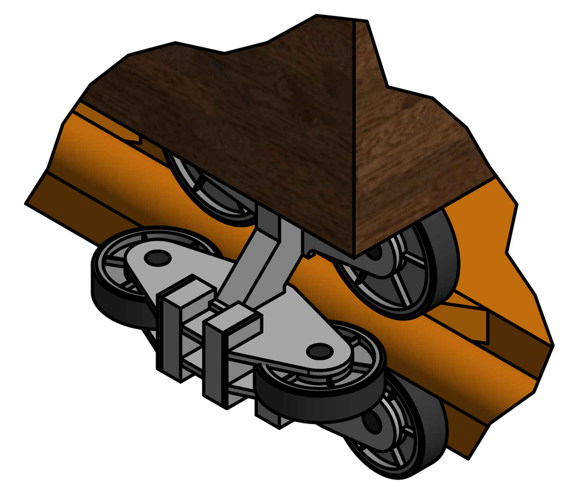
This drawing serves as a realistic depiction of where and how the general body and locations of parts of the mechanism are located. The size and scale of parts are subject to change based on more accurate calculations of forces and necessary resistances.

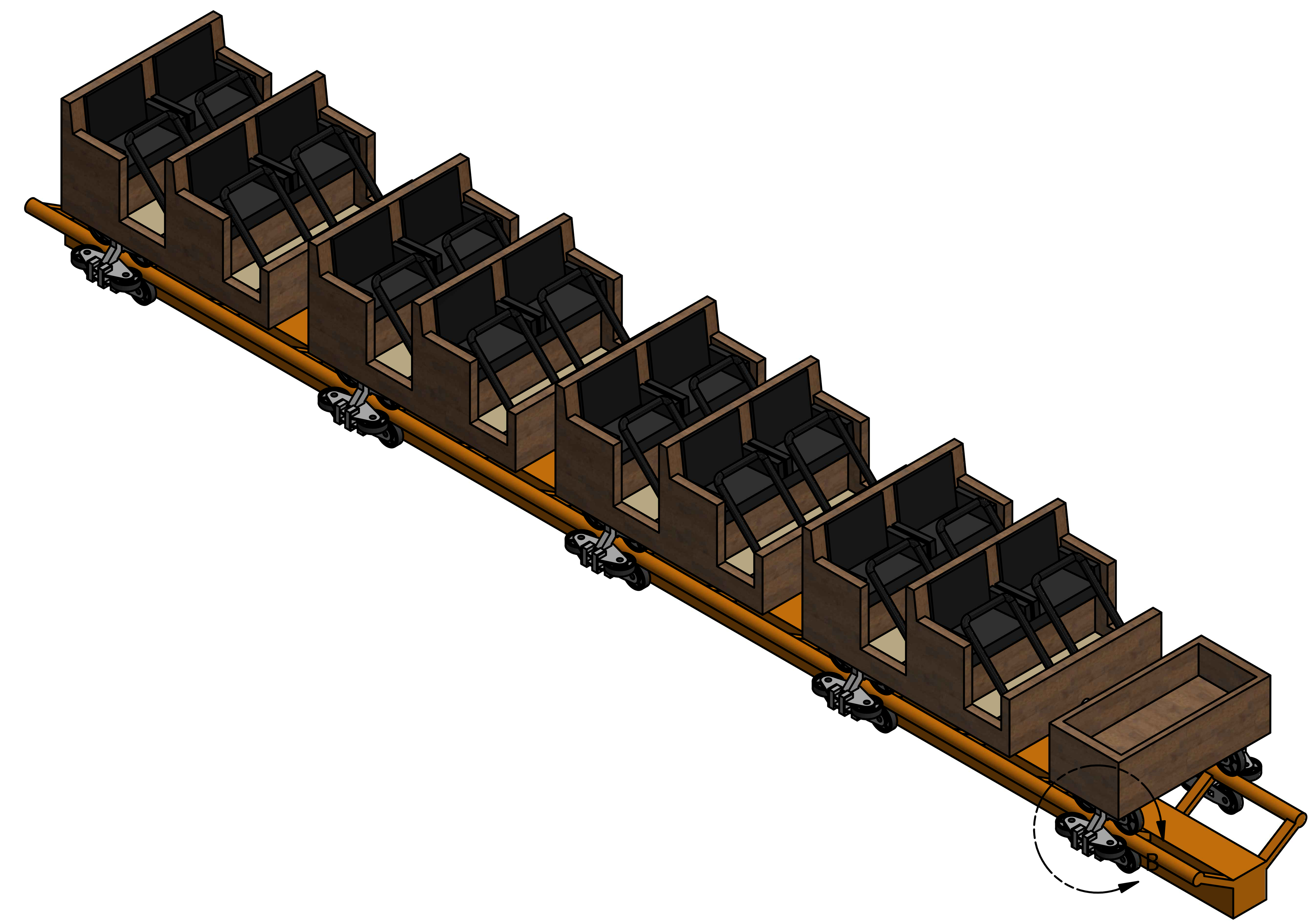
DRAWN TEAM 10	5/1/2019	
CHECKED		TITLE
QA		Glacier Gorge Novel Technology
MFG		
APPROVED		SIZE D · DWG NO Mechanism and Chassis · REV
		SCALE .03 · SHEET 1 OF 1

DETAIL A
SCALE .1:1



DETAIL B
SCALE .1:1





Dimensions of the wheels should be designed to withstand the force of 22000/n Newtons, where n=number of wheels, as shown in the provided proof. For wheel size and scale, this depends on the type of material used and the amount of friction that is desired.

The chassis is designed based off of images from real-life roller coasters, but does not necessarily adhere to exact dimensions.

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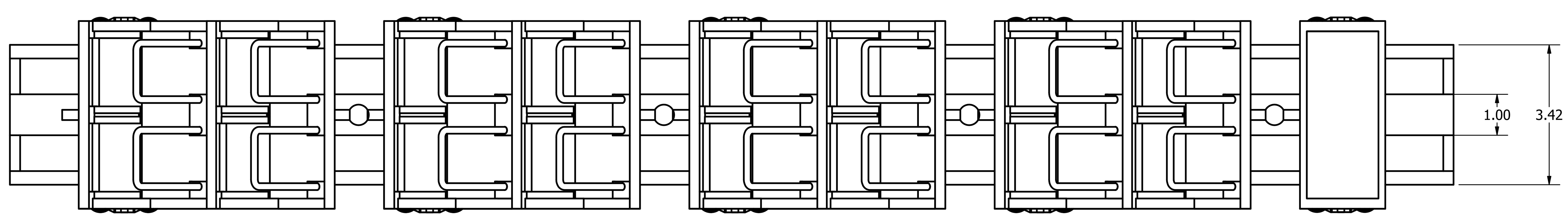

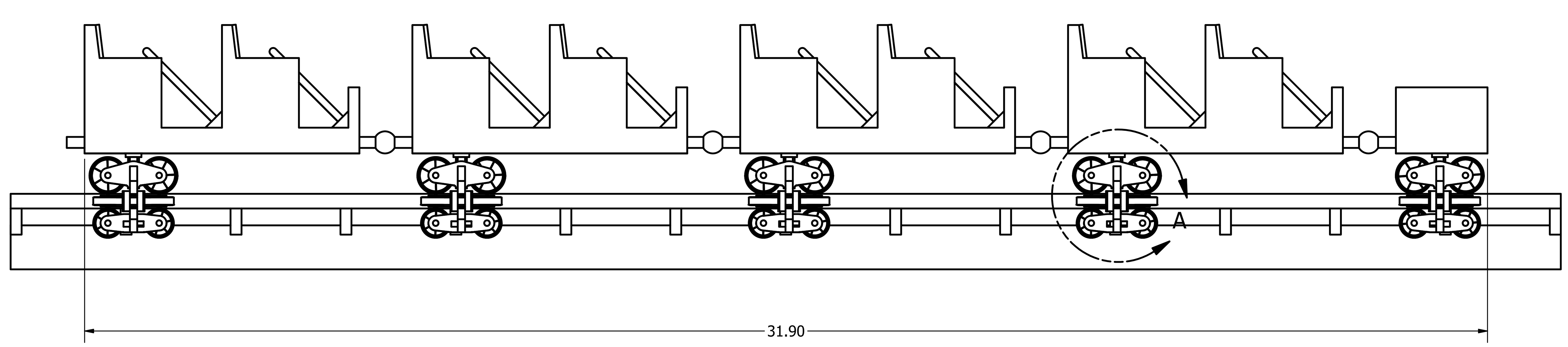

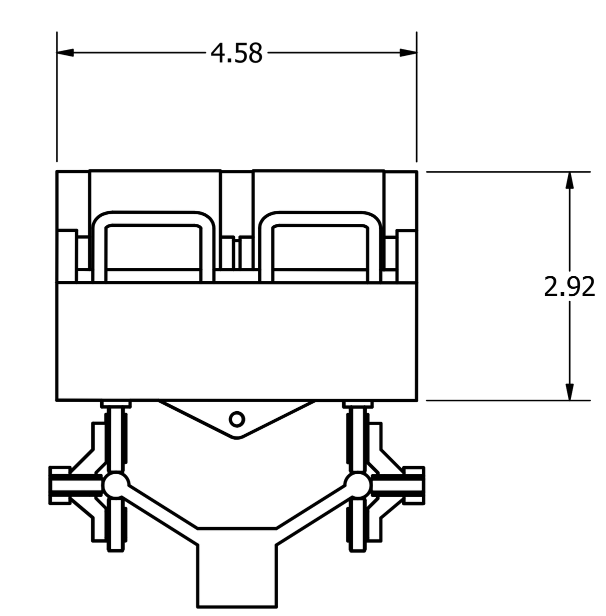

DRAWN TEAM 10	5/1/2019	
CHECKED		TITLE
QA		Glacier Gorge Novel Technology
MFG		
APPROVED		SIZE D · DWG NO Coaster and Track · REV
		SCALE .04 · SHEET 1 OF 1

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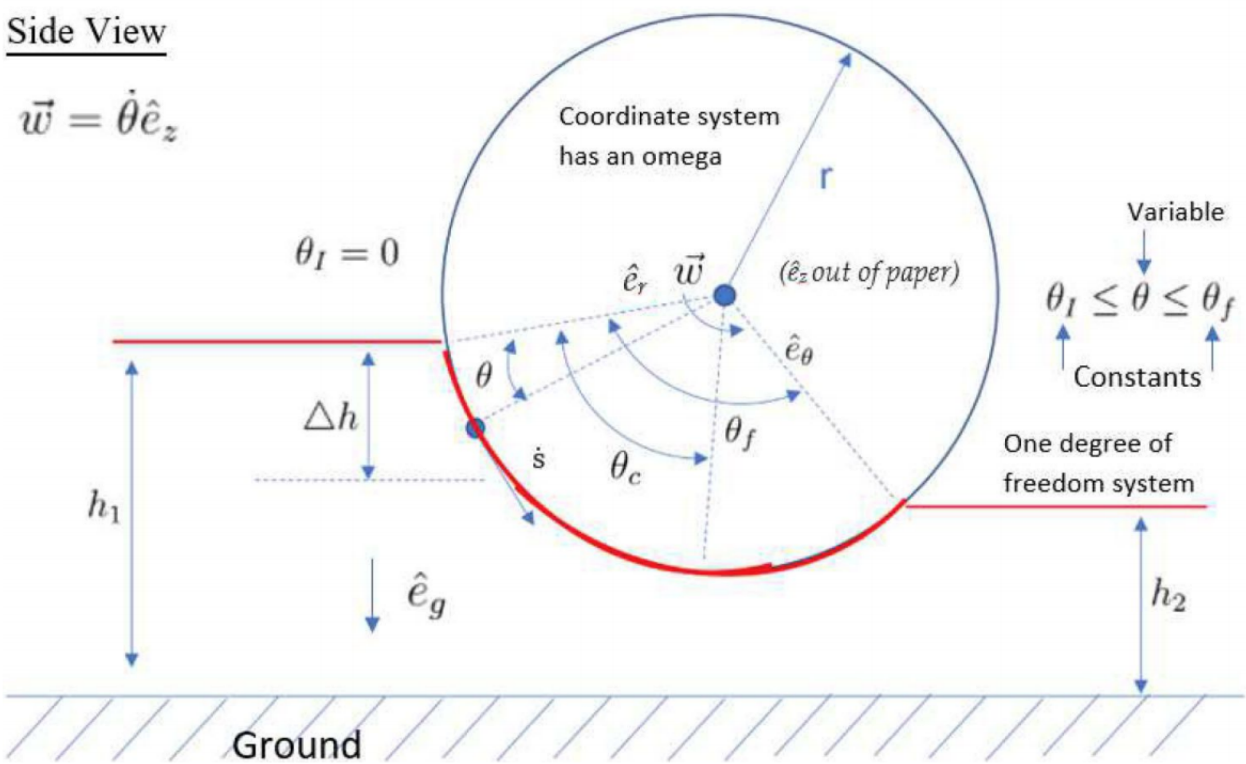
Technical Analysis

Team 10

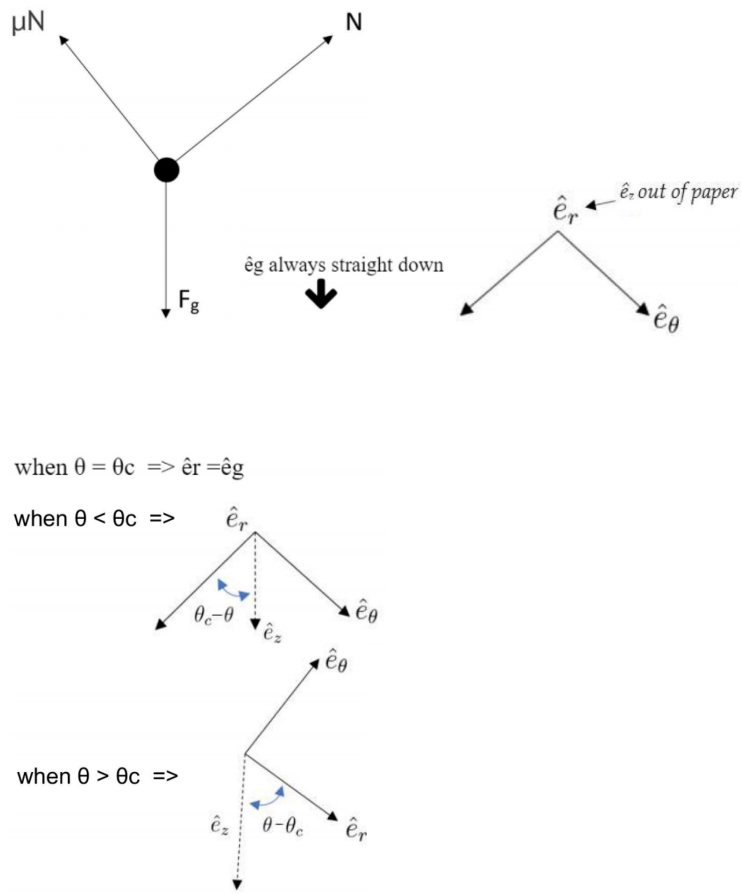
May 3, 2019



Rollercoaster Track Free Body Diagram

Position Vector:
$\vec{r} = r\hat{e}_r$
$\dot{\vec{r}} = r\dot{\vec{e}}_r = r(wx\hat{e}_r) = r(\dot{\theta}\hat{e}_z x\hat{e}_r) = r\dot{\theta}\hat{e}_\theta$
$\ddot{\vec{r}} = r\ddot{\theta}\hat{e}_\theta + r\dot{\theta}\dot{\hat{e}}_\theta = r\ddot{\theta}\hat{e}_\theta + r\dot{\theta}(\dot{\theta}\hat{e}_z x\hat{e}_\theta)$
$\ddot{\vec{r}} = r\ddot{\theta}\hat{e}_\theta - r\dot{\theta}^2\hat{e}_r$

Let's examine the forces on or cart using point mass analysis for simplicity.



Forces on Cart

We need to project $\hat{e}_z$ onto $\hat{e}_\theta$ and $\hat{e}_r$ to exhibit this behavior.

$\hat{e}_z = cos(\theta_c - \theta)\hat{e}_r + sin(\theta_c - \theta)\hat{e}_\theta$

Careful examination of cases reveals that this expression for the projection works beautifully. Let's write the sum of the forces now.

$\vec{F} = -\mu N\hat{e}_\theta - N\hat{e}_r + mg\hat{e}_z = -\mu N\hat{e}_\theta - N\hat{e}_r + mg[cos(\theta_c - \theta)\hat{e}_r + sin(\theta_c - \theta)\hat{e}_\theta]$
$\vec{F} = [mgsin(\theta_c - \theta) - \mu N]\hat{e}_\theta + [mgcos(\theta_c - \theta) - N]\hat{e}_r$

Now we use: $\vec{F} = \frac{d\vec{p}}{dt} = m\ddot{\vec{r}}$ in this case $(F = ma)$

This gives us 2 equations of motion

In θ direction:
$mr\ddot{\theta} = mgsin(\theta_c - \theta) - \mu N$

In r direction:
$-mr\dot{\theta}^2 = mgcos(\theta_c - \theta)$

We now have a system of coupled differential equations that is nonlinear. We will use numerical methods to solve this for initial conditions.

θ - direction: $\ddot{\theta} = \frac{g}{r}sin(\theta_c - \theta) - \frac{\mu N}{mr}$
r - direction: $\dot{\theta}^2 = -\frac{g}{r}cos(\theta_c - \theta) + \frac{N}{mr}$

To eliminate N:

$\dot{\theta}^2 + \frac{g}{r}cos(\theta_c - \theta) = \frac{-\ddot{\theta} + \frac{g}{r}sin(\theta_c - \theta)}{m}$
$u\dot{\theta}^2 + \frac{mg}{r}cos(\theta_c - \theta) = \frac{g}{r}sin(\theta_c - \theta) - \ddot{\theta}$

This allows us to solve for $\theta, \dot{\theta}$, and $\ddot{\theta}$ response.

Using θ-response, we will solve for N.

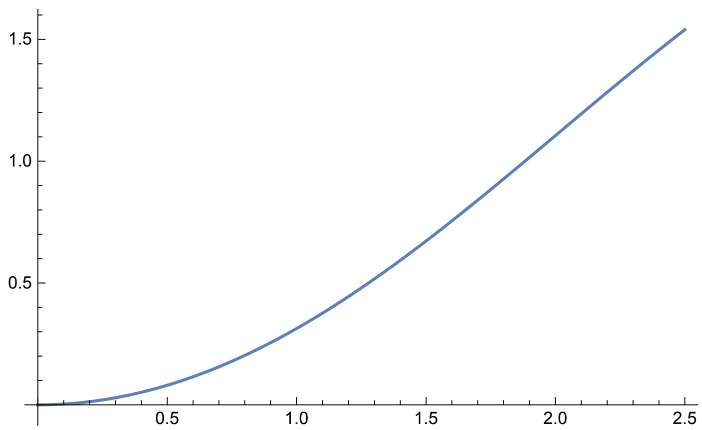
$N = [\dot{\theta}^2 + \frac{g}{r}cos(\theta_c - \theta)](mr)$

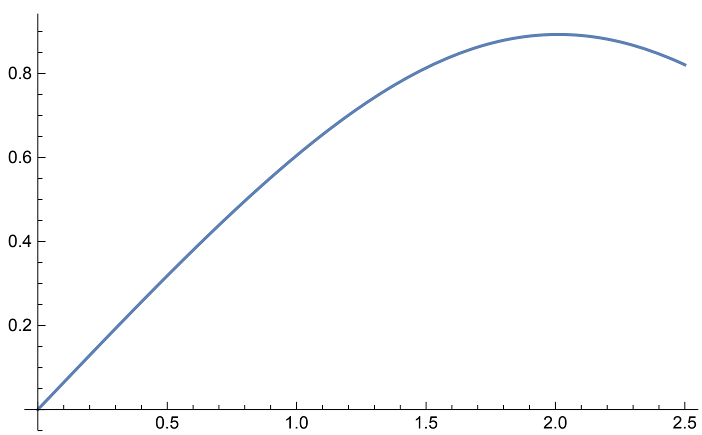
$N = 22,000$

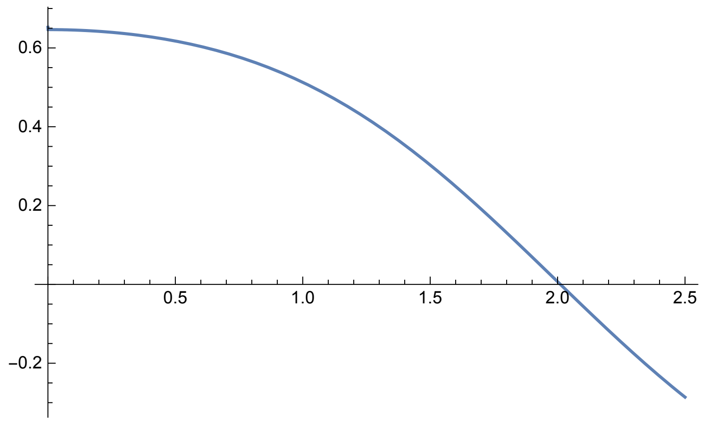
The $22,000$ Newton force is distributed by all 4 wheels for a total of 5500 Newtons per wheel.

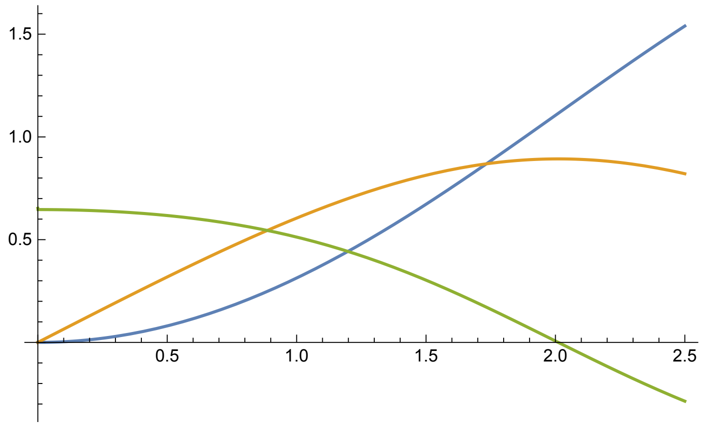
```
Clear["Global`*"]
ycrit = 70 * Degree;
yfinal = 90 * Degree;
g = 9.81;
r = 14;
M = 5216;
u = .05;
eqns = {y''[t] == g * Sin[ycrit - y[t]] / r - u * y'[t] * y'[t] - u * g * Cos[ycrit - y[t]] / r,
   y[0] == 0, y'[0] == 0};
solution = NDSolve[eqns , {y, x}, {t, 0, 2.5}]
Plot[y[t] /. solution, {t, 0, 2.5}]
Plot[y'[t] /. solution, {t, 0, 2.5}]
Plot[y''[t] /. solution, {t, 0, 2.5}]
Plot[Evaluate[{y[t], y'[t], y''[t]} /. solution], {t, 0, 2.5}]
Plot[Evaluate[(y'[t] * y'[t] + g * Cos[ycrit - y[t]] / r) * M * r /. solution], {t, 0, 2.1}]
```

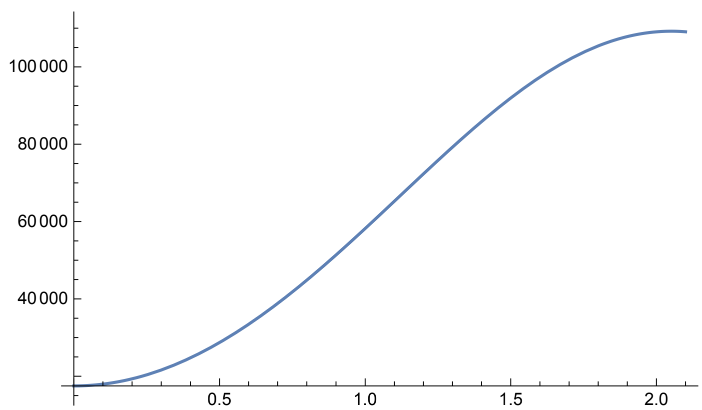
```
{{y → InterpolatingFunction[ Domain: {{0., 2.5}} Output: scalar ], x → x}}
```











1st plot = This is a plot of angle vs time all in SI units (radians and seconds)
2nd plot = This is a plot of angular velocity vs time all in SI units (radians/s and seconds)
3rd plot = This is a plot of angular acceleration vs time all in SI units (radians/s/s and seconds)
4th plot = Plots 1, 2, and 3 on top of each other
5th plot = Plot of normal force vs time: the peak normal force lags behind the peak angular velocity (bottom of the circle) a bit. In other words, peak normal force happens a bit after the lowest point on the track. It can be seen that the maximum is around 21,500 Newtons total. Number N wheels will each experience a force that is (21,500/N) Newtons. This force is a function of time and is expressed in this 5th plot for the train going through this part of the track.